

The Improbable Differential Attack: Cryptanalysis of Reduced Round CLEFIA

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 - Introduction
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- And others (Higher-order Differential, Boomerang,...)

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Improbable Differential	p	p_0	$p_0 < p$

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Caution

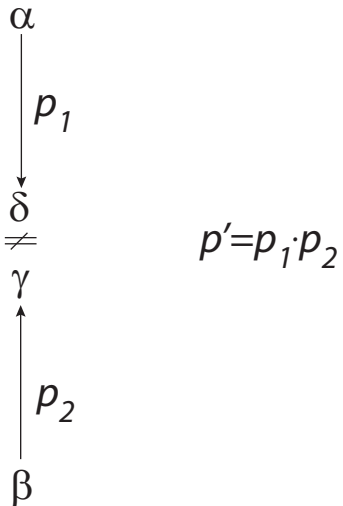
If there are nontrivial differentials from α to β , p_0 becomes bigger than $p \cdot (1 - p')$.

Two Techniques to Obtain Improbable Differentials

Two methods to obtain improbable differentials:

- 1 Use two differentials that miss in the middle with high probability (almost miss in the middle technique)
- 2 Expand impossible differentials to improbable differentials by adding a differential to the top and/or below the impossible differential (expansion technique)

Almost Miss-in-the-Middle Technique

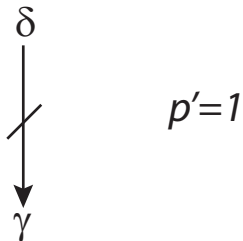


Improbable Differentials

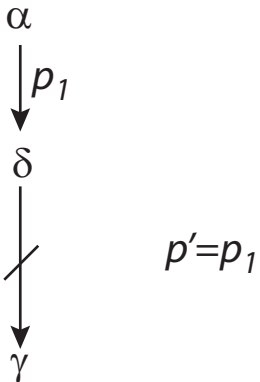
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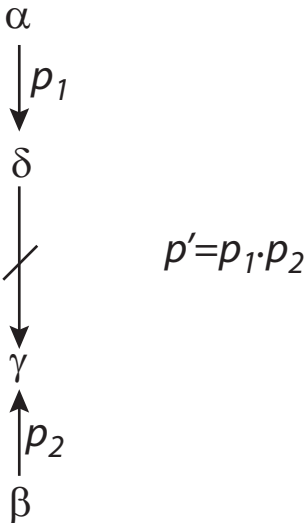
Improbable Differentials from Impossible Differentials



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Pros and Cons of the Expansion Method

Pros:

- Longer differentials
- Attack on more rounds

Cons:

- Data complexity increases (because p_0 increases)
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Previous attacks where $p_0 < p$

Early examples of improbable differential attack:

- J. Borst, L. Knudsen, V. Rijmen: "Two Attacks on Reduced IDEA"
- L. Knudsen, V. Rijmen: "On the Decorrelated Fast Cipher (DFC) and Its Theory"

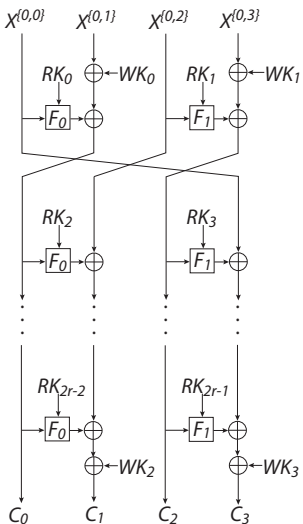
CLEFIA

- Developed by Sony in 2007
- *Clef* means *key* in French.
- Block length: 128 bits
- Key lengths: 128, 192, and 256 bits
- Number of rounds: 18, 22, or 26
- Previous best attacks: Impossible differential attacks on 12, 13, 14 rounds for 128, 196, 256-bit key lengths by Tsunoo et al.
- We converted these attacks to improbable differential attacks using the expansion technique
- Current best attacks: Improbable differential attacks on 13, 14, 15 rounds for 128, 196, 256-bit key lengths

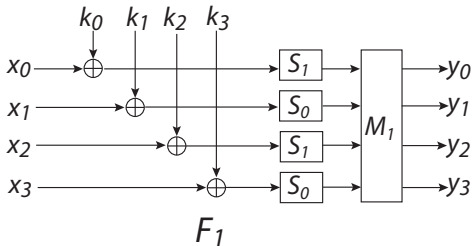
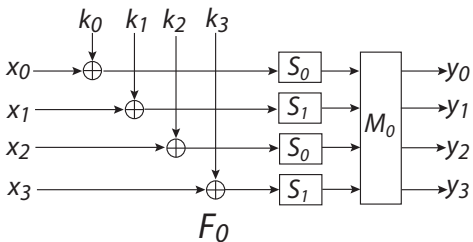
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CLEFIA: Encryption Function



CLEFIA: F_0 and F_1 Functions



10-round Improbable Differential

We will use the following two 9-round impossible differentials that are introduced by Tsunoo et al.,

$$\begin{aligned} [0_{(32)}, 0_{(32)}, 0_{(32)}, [X, 0, 0, 0]_{(32)}] &\not\rightarrow_{9r} [0_{(32)}, 0_{(32)}, 0_{(32)}, [0, Y, 0, 0]_{(32)}] \\ [0_{(32)}, 0_{(32)}, 0_{(32)}, [0, 0, X, 0]_{(32)}] &\not\rightarrow_{9r} [0_{(32)}, 0_{(32)}, 0_{(32)}, [0, Y, 0, 0]_{(32)}] \end{aligned}$$

where $X_{(8)}$ and $Y_{(8)}$ are non-zero differences.

10-round Improbable Differential

We obtain 10-round improbable differentials by adding the following one-round differentials to the top of these 9-round impossible differentials,

$$\begin{aligned} [[\psi, 0, 0, 0]_{(32)}, \zeta_{(32)}, 0_{(32)}, 0_{(32)}] &\rightarrow_{1r} [0_{(32)}, 0_{(32)}, 0_{(32)}, [\psi, 0, 0, 0]_{(32)}] \\ [[0, 0, \psi, 0]_{(32)}, \zeta'_{(32)}, 0_{(32)}, 0_{(32)}] &\rightarrow_{1r} [0_{(32)}, 0_{(32)}, 0_{(32)}, [0, 0, \psi, 0]_{(32)}] \end{aligned}$$

which hold when the output difference of the F_0 function is ζ (resp. ζ') when the input difference is $[\psi, 0, 0, 0]$ (resp. $[0, 0, \psi, 0]$).

13-round Improbable Differential Attack

We choose ψ and corresponding ζ and ζ' depending on the difference distribution table (DDT) of S_0 in order to increase the probability of the differential. In this way we get $p' \approx 2^{-5.87}$.

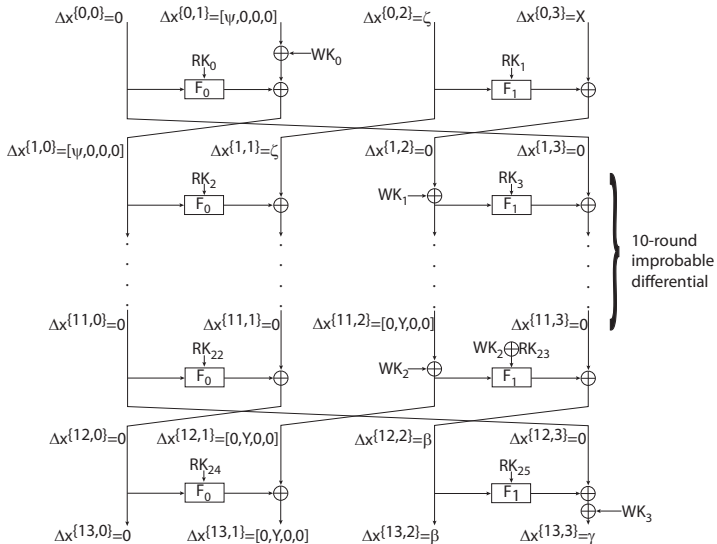
We put one additional round on the plaintext side and two additional rounds on the ciphertext side of the 10-round improbable differentials to attack first 13 rounds of CLEFIA that captures RK_1 , $RK_{23,1} \oplus WK_{2,1}$, RK_{24} , and RK_{25} .

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Table: Comparison of Tsunoo et al.'s impossible attack with the expanded improbable attack

Rounds	Attack Type	Key Length	Data Complexity	Time Complexity	Memory (blocks)	Success Probability
12	Impossible	128	$2^{118.9}$	2^{119}	2^{73}	-
13	Improbable	128	$2^{126.83}$	$2^{126.83}$	$2^{101.32}$	%99

14 and 15-round Improbable Differential Attacks

By using the similar expansion technique, we can apply improbable differential attack on

- 14-round CLEFIA when the key length is 192 bits
- 15-round CLEFIA when the key length is 256 bits

Conclusion

We provided

- 1 a new cryptanalytic technique called *improbable differential attack* where a differential holds with less probability when tried with the correct key
- 2 two techniques to obtain improbable differentials
- 3 data complexity estimates for improbable differential attacks
- 4 state of art attacks on the block cipher CLEFIA

Conclusion

THANK YOU FOR YOUR
ATTENTION